1901MNRAS..62...17H

(2) Beyond the mass μ we have

$$r_2=r_1+1\ ;\ \frac{\partial r_1}{\partial x}=\frac{\partial r_2}{\partial x}$$

$$\therefore\ \frac{\partial\Omega}{\partial r_1}+\frac{\partial\Omega}{\partial r_2}=0$$

$$\therefore\ \mu\left(\frac{1}{r_1^2}-r_1\right)=\nu\left(r_2-\frac{1}{r_2^2}\right)$$

Hence clearly $r_1<1$. In this case

$$t=\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\cdot\mu\left(r_1-\frac{1}{r_1^2}\right)$$

and is negative because $r_1<1<r_2$, and therefore the first factor is positive and the last negative. The proof that t is negative in the case of the centre of libration lying beyond the mass ν can be inferred from the latter, as there is no restriction on the ratio of the masses μ and ν. Now we have

$$r=\frac{\partial^2\Omega}{\partial x^2}=\mu\left(1+\frac{2}{r_1^3}\right)+\nu\left(1+\frac{2}{r_2^3}\right)$$

which is necessarily positive. Hence always for the three points considered

$$rt-s^2<0$$

and therefore the two roots of the quadratic in m^2 are of opposite signs. I have thought it worth while to give a definite proof of this fact because on this point Dr. Charlier says : "That is what I have found through numerical calculation for different values of μ, an algebraic demonstration of this fact seeming to be somewhat complicated."

University Observatory, Oxford :
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A New Method of Interpolation. By T. C. Hudson, B.A.

1. Bessel's interpolation formula is

$$u_n-u_0=n\Delta'_{\frac{1}{2}}+\frac{n(n-1)}{1\cdot2}\,\frac{\Delta_0''+\Delta_1''}{2}+\frac{n(n-1)(n-\frac{1}{2})}{1\cdot2\cdot3}\,\Delta_{\frac{1}{2}}'''$$
$$+\frac{(n+1)n(n-1)(n-2)}{1\cdot2\cdot3\cdot4}\,\frac{\Delta_0^{\text{iv}}+\Delta_1^{\text{iv}}}{2}+\ .\ .\ .\qquad(1)$$

2. As it stands this formula is partially symmetrical with respect to the middle of the interval ; but there is no symmetry

1901MNRAS..62...17H

in the first three terms, namely, u_n, u_0 and $n\,\Delta'_{\frac{1}{2}}$, and the symmetry of the last three does not present itself obviously to the eye. If, however, we replace

$$\begin{array}{llllll}\dfrac{u_0+u_1}{2}, & \dfrac{1}{1\,.\,2}\,\dfrac{\Delta_0''+\Delta_1''}{2}, & \dfrac{1}{1\,.\,2\,.\,3\,.\,4}\,\dfrac{\Delta_0^{iv}+\Delta_1^{iv}}{2}, & \Delta_{\frac{1}{2}}', & \Delta_{\frac{1}{2}}''', & \tfrac{1}{2}-n, u_n \\ \text{by M}, & \text{M}'', & \text{M}^{iv} & , \Delta', & \Delta''', & m, \text{U}_{+m}\end{array}$$

equation (1) becomes

$$\begin{array}{llll}\text{U}_{+m}=\text{M} & -(\tfrac{1}{4}-m^2)\text{M}'' & +(\tfrac{1}{4}-m^2)(\tfrac{9}{4}-m^2)\text{M}^{iv}- \ldots \\ & -m\Delta' & +m(\tfrac{1}{4}-m^2)\dfrac{\Delta'''}{6} & - \ldots\end{array}$$

(a modification of Bessel's formula due, it appears, to Newton) which may be written

$$\text{U}_{+m}=[\text{E}]+[\text{O}]$$

where $$[\text{E}]=\text{M}-(\tfrac{1}{4}-m^2)\text{M}''+(\tfrac{1}{4}-m^2)(\tfrac{9}{4}-m^2)\text{M}^{iv}- \ldots$$

containing only *even* differences and *even* powers of m,

and $$[\text{O}]=-m\Delta'+m(\tfrac{1}{4}-m^2)\frac{\Delta'''}{6}- \ldots$$

containing only *odd* differences and *odd* powers of m.

In the E equation it will be seen that M takes its place naturally with M$''$, M^{iv}, &c., its coefficient obeying the same law as theirs, and that the original quantities may be justly regarded as belonging to the family of even differences.

3. Changing the sign of m, and observing that U_{-m} is merely u_{1-n}, we have

$$\text{U}_{-m}=[\text{E}]-[\text{O}].$$

By these substitutions we have now arrived at a pair of formulæ

$$\left\{\begin{array}{l}\text{U}_{+m}=[\text{E}]+[\text{O}] \\ \text{U}_{-m}=[\text{E}]-[\text{O}]\end{array}\right\} \quad . \; . \; . \; . \; . \; . \; . \quad (2)$$

which possesses the practical, economic advantage of being symmetrical with respect to the middle of the interval.

4. The advantage consists in this, that when once [E] and [O] have been computed for the first half of the interval, and their sum has given the *first* half of the new quantities (u_n or U_{+m}), then their difference gives the *second* half without further computation. This means a saving of half the labour involved in a great part of the work of systematic interpolation. Moreover, the more new quantities we have to interpolate between the old data, the greater (relatively) becomes this economy. In particular, to take the opposite extreme, in the simplest case, that of "bisection," where we should want only midway quantities, we

1901MNRAS..62...17H

observe that m, [O], and the difference between the formulæ (2) would all vanish together—likewise the economy.

But in the actual case of the work of deriving the Moon's hourly places for the *Nautical Almanac* from twelve-hourly data throughout a whole year, the degree of subdivision of the original interval into twelve new ones is so great that the aforesaid economy becomes an important consideration.

5. For this reason I found it worth while, a few years ago, to make new tables for the interpolation of the Moon's hourly R.A. and Dec., in which case (known variously as "interpolation to twelfths" and as "interval twelve") we have the following values :—

of $12n$.	viz.	1 11	2 10	3 9	4 8	5→ 7←	6
of $12m$	,,	{± 5	± 4	± 3	± 2	± 1⇄	0
of the coefficients of							
$\frac{\Delta'}{12}$ in [O]	,,	∓ 5	∓ 4	∓ 3	∓ 2	∓ 1⇄	0
$\frac{1}{6}\,\frac{\Delta'''}{12^3}$ in [O]	,,	±55	±80	±81	±64	±35⇄	0
$\frac{1}{144}\left(M''-\frac{317}{144}M^{iv}\right)$ in [E]	,,	−11	−20	−27	−32	−35⇄	−36
and of the residual coefficients of							
$\frac{1}{2}\,\frac{M^{iv}}{12^3}$ in [E]	viz.	−33	−30	− 9	+16	+35⇄	+42

6. With regard to these tabulated coefficients it may be noted

(*a*) That no other interpolation formula gives such simple coefficients as it stands, nor could it be made to do so by any other modification without seriously impairing the simplicity of treatment of the original differences, and also introducing additional sources of cumulative error.

(*b*) That the second coefficient of $\frac{M''}{144}$ is -20, *i.e.* the second coefficient of $\frac{\text{mean } \Delta''}{144}$ is -10. This is a most happy coincidence, and it means that the second member of the series of Δ'' corrections happens to be numerically identical with ten times the new middle δ'', so that we are practically getting two things at once, and we have an opportunity of condensing the corresponding table by making the second column do double duty by acting as vertical argument as well. There are other practical advantages of this coincidence which will appear later. Suffice it here to say

1901MNRAS..62...17H

that it is the Δ'' table that has hitherto caused the most trouble in construction and in use, and that it is this very table which is hereby simplified.

(*c*) That with a view to (i) dispensing as often as possible with Δ^{iv} corrections (as such) without seriously impairing the simplicity of treatment of the original second differences, and (ii) at the same time minimising resulting error (whether individual or mean, neglected or corrected), I have selected the particular device (as in the above table of coefficients) of subtracting $\frac{317}{1728}$ of the mean Δ^{iv} from the mean Δ'', on the grounds that the resulting "residual Δ^{iv} corrections" are generally negligible, that their mean algebraical value throughout an interval is *zero*, and their maximum value is about half a thousandth of the original mean Δ^{iv}, so that these residuals may be tabulated in three or four lines only, and then never referred to except as a matter of fact when the Moon is near a maximum declination, and that declination is also a large one by reason of the position of the Moon's node. Other devices might have been chosen, but at the sacrifice of one or more of these practical advantages. Moreover, further pursuit of this same method (such as entrusting the broad fifth-difference effect to the third-differences) would on the whole in each case rather complicate than expedite the work.

7. In the tables, which are purely *additive*, only such information is given as is necessary and sufficient, (*a*) to produce the end figure of the new quantities, and (*b*) to enable the calculator to fill in the rest with ease. By this economy the tables are reduced to very small compass.

Comparisons of the Geocentric Places of Uranus, Neptune, and the Sun, calculated from Newcomb's Tables, with their Places calculated from Le Verrier's Tables, for the Year 1904. By A. M. W. Downing, D.Sc., F.R.S.

This paper is a continuation of one printed in *Monthly Notices*, vol. lix., No. 10. The *Nautical Almanac* for 1904 is the first in which Newcomb's Tables of *Uranus* and *Neptune* are used, and a comparison is made at convenient intervals throughout the year between the places there given for Greenwich noon and the corresponding places computed from those given in the *Connaissance des Temps* for 1904 for Paris noon.

I have also added a comparison of the Sun's places, deduced in the same manner, at intervals of eight days throughout the year.